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Design of a vertical blade horizontal wind rotor

Ming Kwang Tsu
Iowa State College

DESIGN OF A VERTICAL BLADE HORIZONTAL WIND ROTOR

by

Ming Kwang Tsu

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I. INTRODUCTION

Many important machines today are developed by imitating the motion of animals. Aeroplane designs are based upon what men have learned from the flight of the bird. The submerging mechanism of a submarine is just another imitation of the fish. It was in 1923 that Miller (15) first observed the phenomenon that the wing tip motion of a sea gull resembles the cycloidal motion. In 1928 Kirsten (13) invented the cycloidal propulsion mechanism for aircraft and boats. With blades parallel to the central shaft and rotating in the fashion of an old type water wheel, Kirsten claimed that the cycloidal propulsion has many advantages over the conventional propeller other than better efficiency. His invention, however, has not been popularly adopted by the present day aircraft builders, mainly because of its excessive weight which greatly handicaps the maneuverability and the load capacity.

Since weight is not an important factor in windmill designing, it seems very promising that this type of machine can be used reversely as a windmill. This thesis is chiefly dedicated to the solution of this problem.

II. REVIEW OF WINDMILL TYPES

The origin of the windmill is obscure. According to the information given by Beckmann (3), Bennett and Elton (4), Ball (2), and Vorolis (22), it seems to have been in Persia. It is recorded that the Khalif Omar was murdered in A. D. 643 by a Persian carpenter "skilled in the construction of windmills" (12). The windmill was first introduced to Europe in the twelfth century.

A. General Types

The general types of windmills are those which have the rotational plane of the sails normal to the direction of the wind. The earliest windmills of this type were known as the Post and the Tower mills. The Post mill was distinguished by the fact that the whole building, carrying sails, cap and all machinery, was pivoted on a huge timber post to face the wind, while the Tower mill had only the cap to be pivoted.

The earliest windmill sails were frameworks of wood forming flat planes having a constant angle to the windshaft which carried them. Sail cloth was laced in and out of the horizontal bars and made fast at top and bottom. It was reefed by being drawn toward the center like a curtain. This type of sail is still found in Cape Cod, U. S. A. Other early sails had flat boards made up into sections which could be put on or taken off as required. They can still be found in Sweden. The sections were known as "under" (at the tip), "middle", "storm", and "permanent". The use of different sections is determined by the prevailing wind velocities in different seasons of the year.

The primitive sails were improved by giving a twist to their length like a propeller. Smeaton (19) was the first man who investigated this scientifically. The millwrights, however, already had adopted this construction principle to get rid of the "pull" before 1780 without knowing the reason. The angles of sails at different sections from the center as given by Smeaton and others (6) are listed in Table 1.

Table 1. Angles of different sections of sails (5)

	Center 0	6 parts of radius 1	2	3	4	5	Tip 6	Best ratio of tip velocity to wind
Smeaton	-	18	19	18	16	12.5	7	2.7
Forester	-	24	21	18	14	9	3	---
Molesworth	-	22.5	21	18.5	15	10.5	5	2.6

In 1772 Andrew Meikle (6, 11), inventor of the threshing machine, brought out his "spring sail". For the cloth of the common sail he substituted a number of "shutters" or "vanes" of canvas stretched over wire frames and hinged on the side nearest to the windshaft. These shutters were all connected to a common "sail bar" by means of cranks and worked together like the laths of a Venetian blind. The sail bar was controlled by elliptic springs which automatically held the vanes closed in calm winds and opened them in stormy weather.

A safety governing device was thus developed. In addition to his spring sail Andrew Meikle also invented the automatic method of controlling the mill so that the sail would always face the wind. At the rear of the tower mill he mounted a small windmill which would always deliver power transmitted through the worm and gears to turn the tower mill until it was in the best position for power generation.

In 1924 a Dutch millwright, A. J. Dekker (6), won the first prize in a competition organized by De Hollandsche Molen to improve the old-fashioned windmill without spoiling its appearance. His prize was for two improvements, namely, the streamlining of the sail by means of an aluminum sheet airfoil and the reduction of the main shaft friction by using a roller bearing. These improvements were tested by the State Aeronautical Research Institute, first, in a wind tunnel, and then on a windmill, against the normal type of sail. It was found that the Dekker roller bearing reduced the shaft neck friction to one-seventh of the old value. With the old type of sail at 15 revolutions per minute in a stiff wind, 50 horse power was developed; though 20 percent of the wind was utilized, only 9 to 11 percent was converted to useful work. With the Dekkerized sail 74 percent of the wind was utilized and 34 to 52 percent was converted into useful work. If the above testing data are correct, the Dekkerized sail would be three times as efficient as the old type.

Since 1925 many windmills in Europe, especially in Holland and Belgium, have been Dekkerized.

The American type of windmill was invented by John Burnham (16) in the middle of the nineteenth century century. The distinction between the older four-arm type and the American type lies in the number and disposition of the sails, which in the latter are numerous (around 20) and form a comparatively narrow ring of vanes. The American type, besides being more efficient than the old type, has the advantage of cheapness, a more even turning moment, and easier starting. It is now commonly used in the United States for farm pumping purposes.

Until the present decade, data covering windmill design have been meager, and for one hundred and fifty years the rough tests of Smeaton (19) have not been greatly bettered. Fales (7, 8) first applied the wind tunnel method and developed the propeller type windmill. Owing to the fact that this type of windmill has only a few (2 - 4) true propeller type blades of comparatively small pitch angle, it is able to revolve with its blade tip speed many times faster than the wind velocity. It is, therefore, mainly used for electric generation. The armature of the generator can be directly coupled with the windmill shaft without any speeding-up gearing mechanism.

The windmills common in use today can be divided into three types, namely, the Dutch, the American, and the propeller types. Their characteristics are summarized in Table 2.

Table 2. Common windmill characteristics

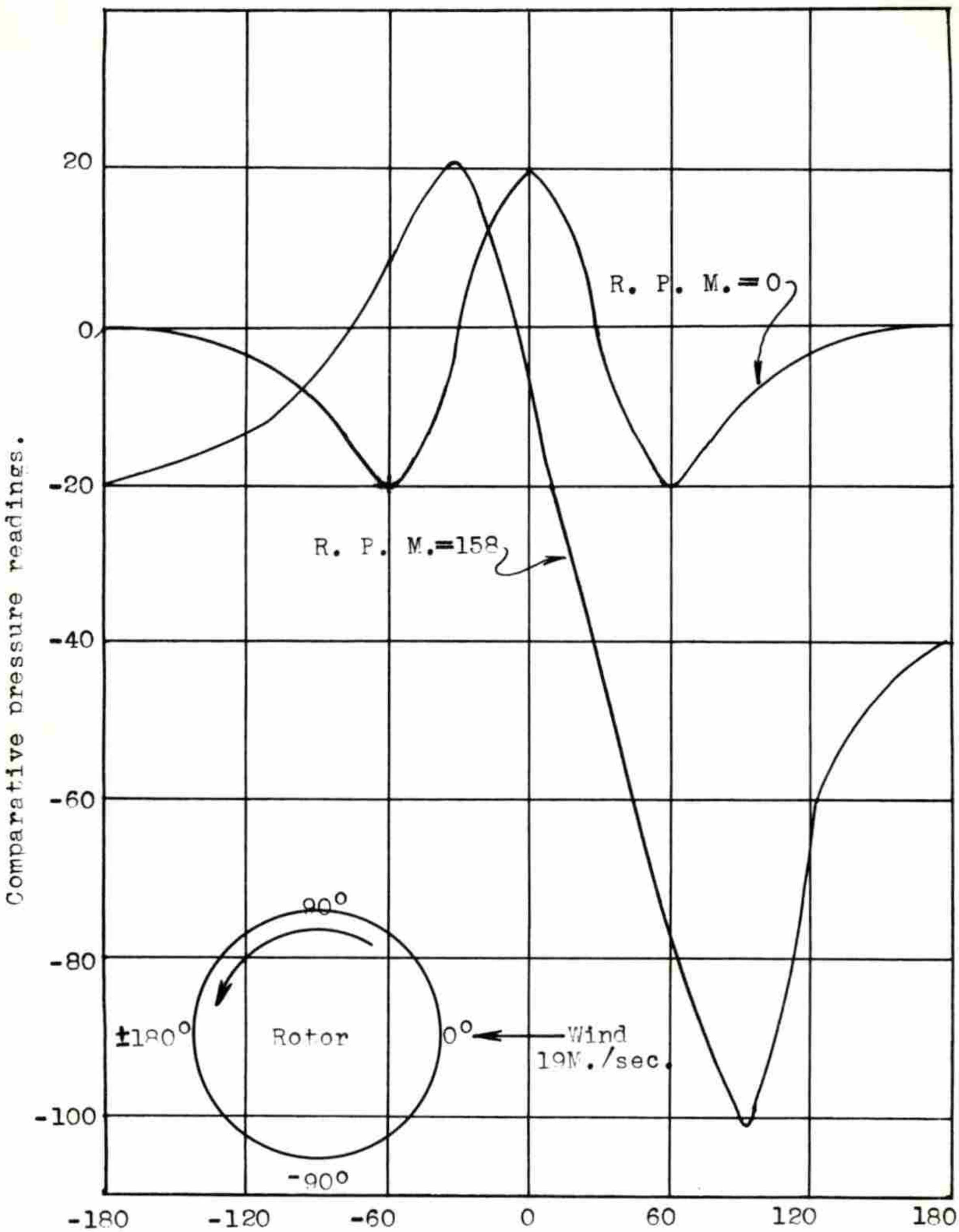
Type	Rotational speed	Ratio of tip speed to wind speed	Efficiency %	Uses
Dutch	Very low	2-3	16	Milling, pumping
American	Medium	1	30	Pumping
Propeller	High	6-7	42	Electric generation

B. Special Types

Besides the above mentioned common types, several other types radically different in appearance are discussed below:

<u>Flettner rotor (24)</u>

When a rotating cylinder is placed in the moving air with its longitudinal axis perpendicular to the wind direction, it produces a force nearly at a right angle to the direction of flow very similar to aeroplane wings. The distribution of pressure around the cylinder is shown in Figure 1. The lift coefficient is much higher than the ordinary airfoils, especially when circular disks are mounted on the ends of the cylinder, (Figure 2). The magnitude of the lift is governed by u/v, i.e., the ratio of rotor speed to wind velocity, (Figure 3). A study of the three curves indicates three



Position around the Flettner's rotor in degrees.

Fig. 1. Distribution of pressure around Flettner's rotor

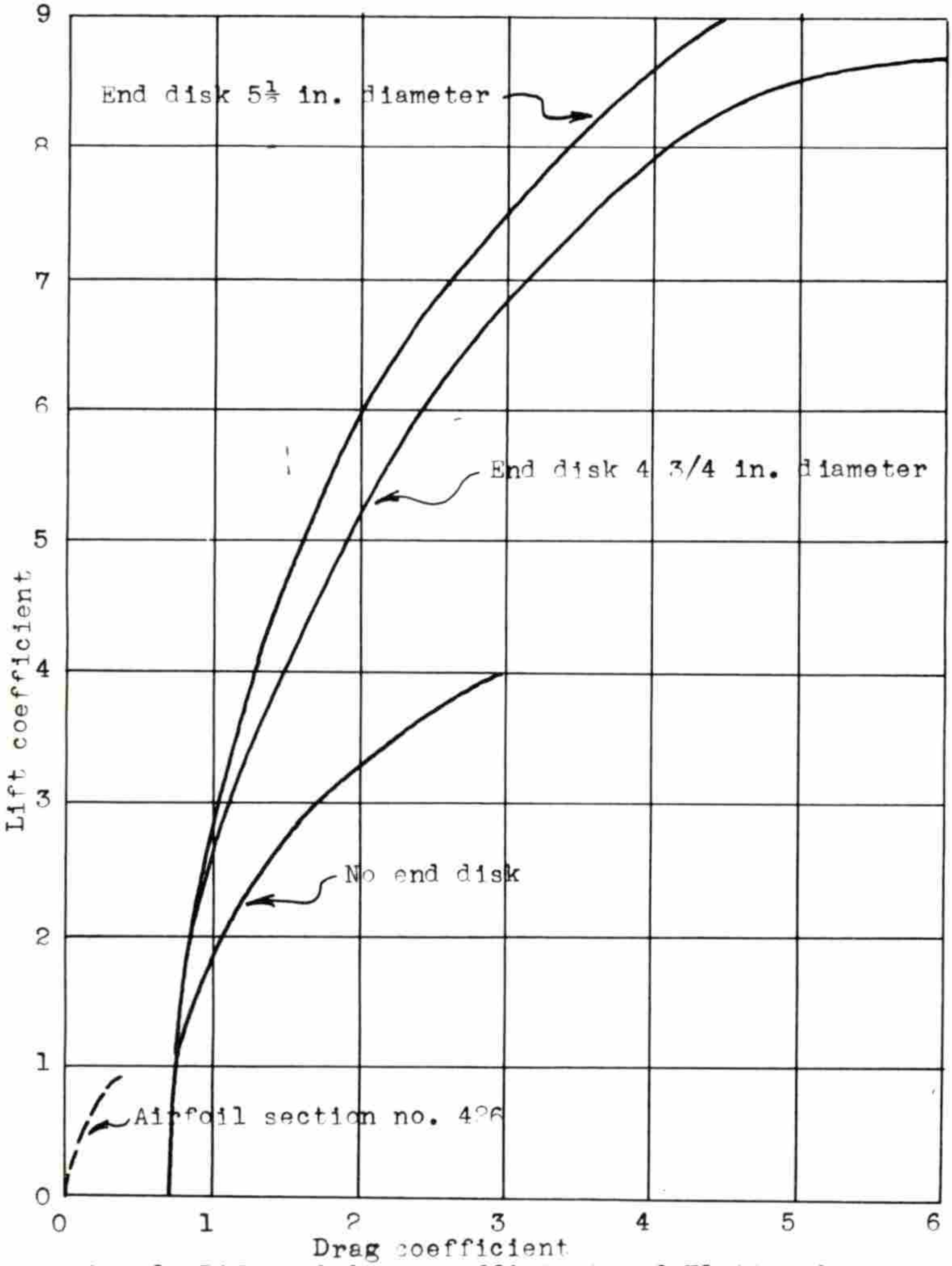


Fig. 2. Lift and drag coefficients of Flettner's rotor

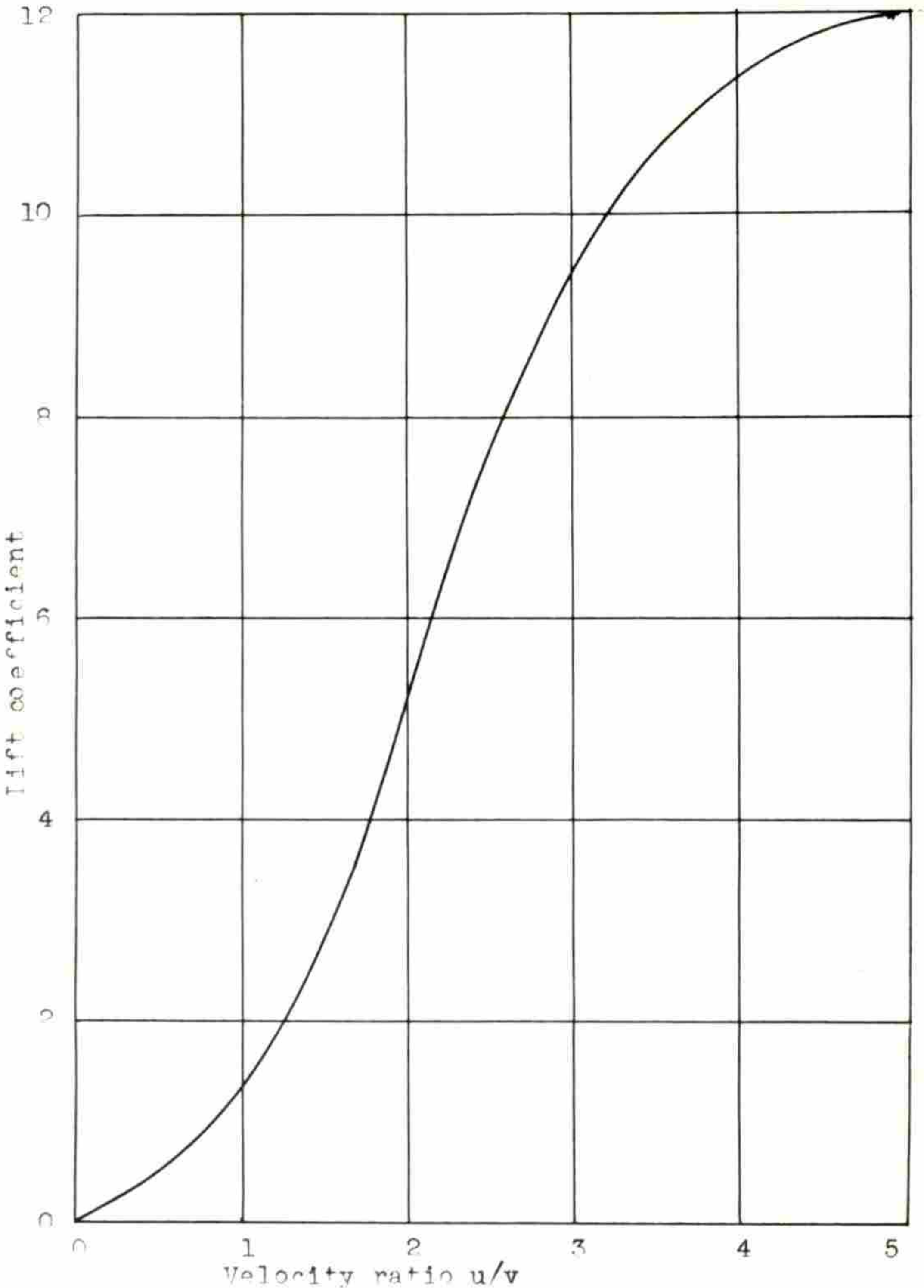


Fig. 3. Effect of velocity ratio to lift coefficient

characteristics:

1. The rotor has a very large lift coefficient compared to ordinary airfoils.

2. Its resistance in the direction of the wind, or the drag, is much larger than the ordinary airfoils, but the ratio of the lift to the drag is relatively high.

3. By varying the speed of rotation the total pressure exerted by the wind upon the rotor or the resultant of the lift and the drag at any wind velocity may be varied between the minimum,when the rotor is not revolving, and the maximum, which depends upon the speed with which the rotor is made to revolve but which in anycase does not exceed the value reached as soon as the peripheral speed becomes a certain multiple of the wind velocity.

4. By laboratory experiments it has been found that the power required to revolve the rotor at the velocity giving maximum lift is less than 10 percent of the energy abstracted from the wind.

One windmill of this type was constructed in 1927 with four Flettner rotors each driven by a small motor mounted inside the rotors to give the rotational speed and spaced like the blades of an ordinary windmill with an over-all diameter of 65 feet and 8 inches. One of the drawbacks of this type of windmill is the complicated rotor driving device which actually limits its use. Willhofft suggested that the small rotor driving motors could be substituted by Savonius rotors mounted on the tips of the Flettner rotors. We have seen in Figure 3 that the highest lift coefficient is reached only when u/v is over 4; also, due to the fact that the rotor diameter is comparatively small, considerable doubt is raised as to whether the Savonius rotor could give a peripheral speed high enough to reach the designated u/v.

Savonius rotor (1, 18)

If an anemometer is dispensed of its arms and the cups are joined together in the fashion shown in Figure 4, a negative pressure will be created at the back side of one cup when it is subjected to the wind. As is obvious in the figure this kind of arrangement is not very good for power generation. However, if semicircular cylinders are used in place of the cups and arranged as shown in Figure 5, the shortcoming of the anemometer is amended, and considerably more power is generated. The Savonius rotor has been tested against an 18-vaned American windmill, and the results are given in Table 3. In another experiment the best number and arrangement of the semicircular vanes were found to be two vanes with their passage 1/5 of the spread. The inventor claimed this kind of wind rotor had the following advantages:

1. More power than the American type for same wind

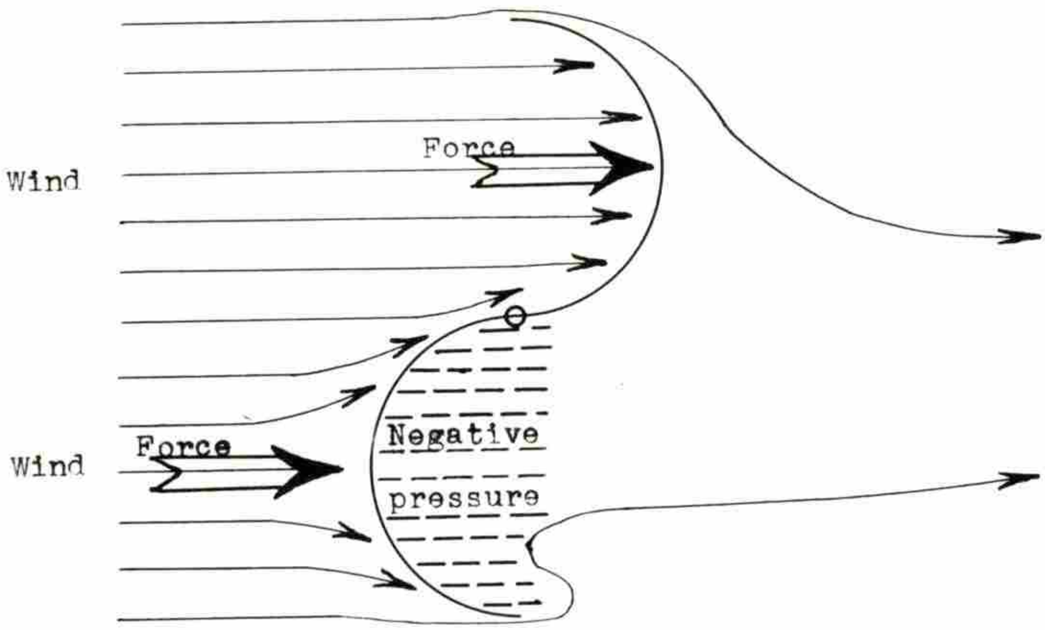


Fig. 4. Pressure distribution around two spherical cups

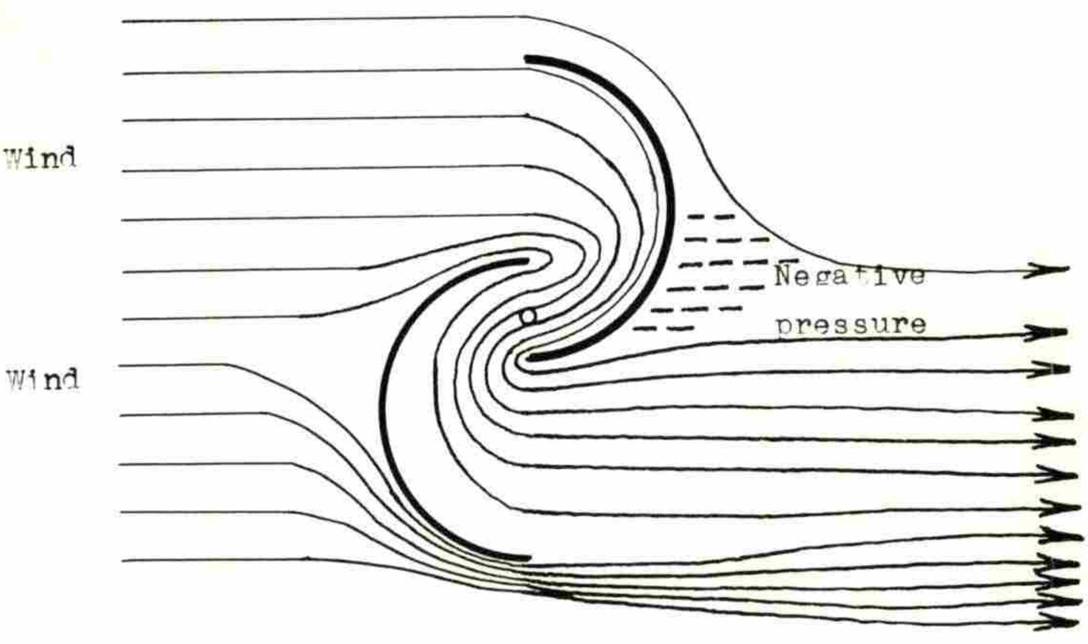


Fig. 5. Pressure distribution around the Savonius rotor

speed and projected area of the wings.

2. Simple construction.

3. Ability to start at low wind velocities.

Table 3. Comparative test of
American windmill and Savonius rotor

Wind rotor	Load	rpm	Work rating	Power ratio
18-vaned windmill	1.3	390	507	75
Savonius (path 1/5 of spread)	1.3	520	676	100
Savonius (path 1/4 of spread)	0.9	705	634	93

Vertical blade horizontal rotor

One windmill of this type was recorded to have been used in China to pump sea water for manufacture of salt (17). A central post some 20 feet long was pivoted at the top and bottom and carried 8 radial arms at both top and bottom, each about 15 feet long. Each pair of arms supported a vertical mast, which carried a sail of cloth between horizontal booms. These were attached to the mast at about their one-third points by loops, and the longer ends were fastened to the following mast, as shown in Figure 6. The sails were arranged so that they could be lowered to avoid destruction when the wind velocity became too great.

From position A to E the wider part of the sail is inward, and the face of the sail makes only a small angle with

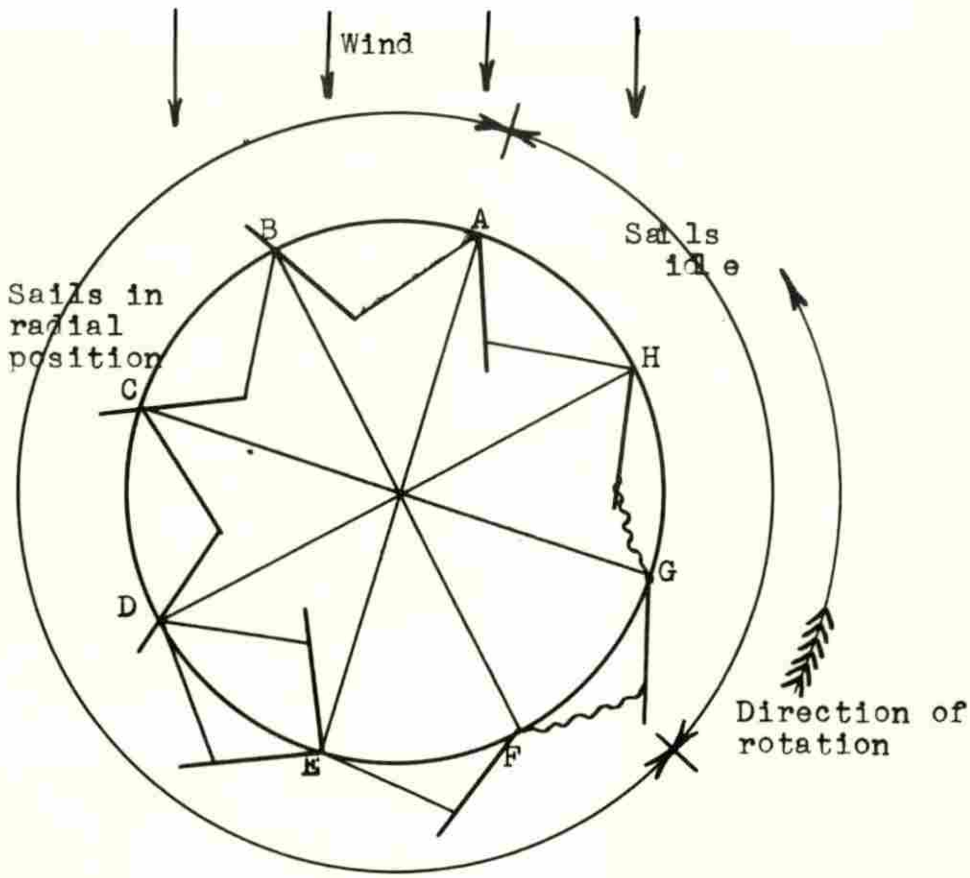


Fig. 6. The mechanism of a Chinese windmill

the radius. As the sail passes position E, the wind catches it on the opposite side and it flops to the other position in which the wider part of the sail is outward. It continues in the position until it reaches a point between F and G, when the sail is again edgewise to the wind and the ropes become slack. From here to the original position the sail is idle and remains parallel to the direction of the wind. It develops 0.58 horse power in a 6-mile-per-hour wind.

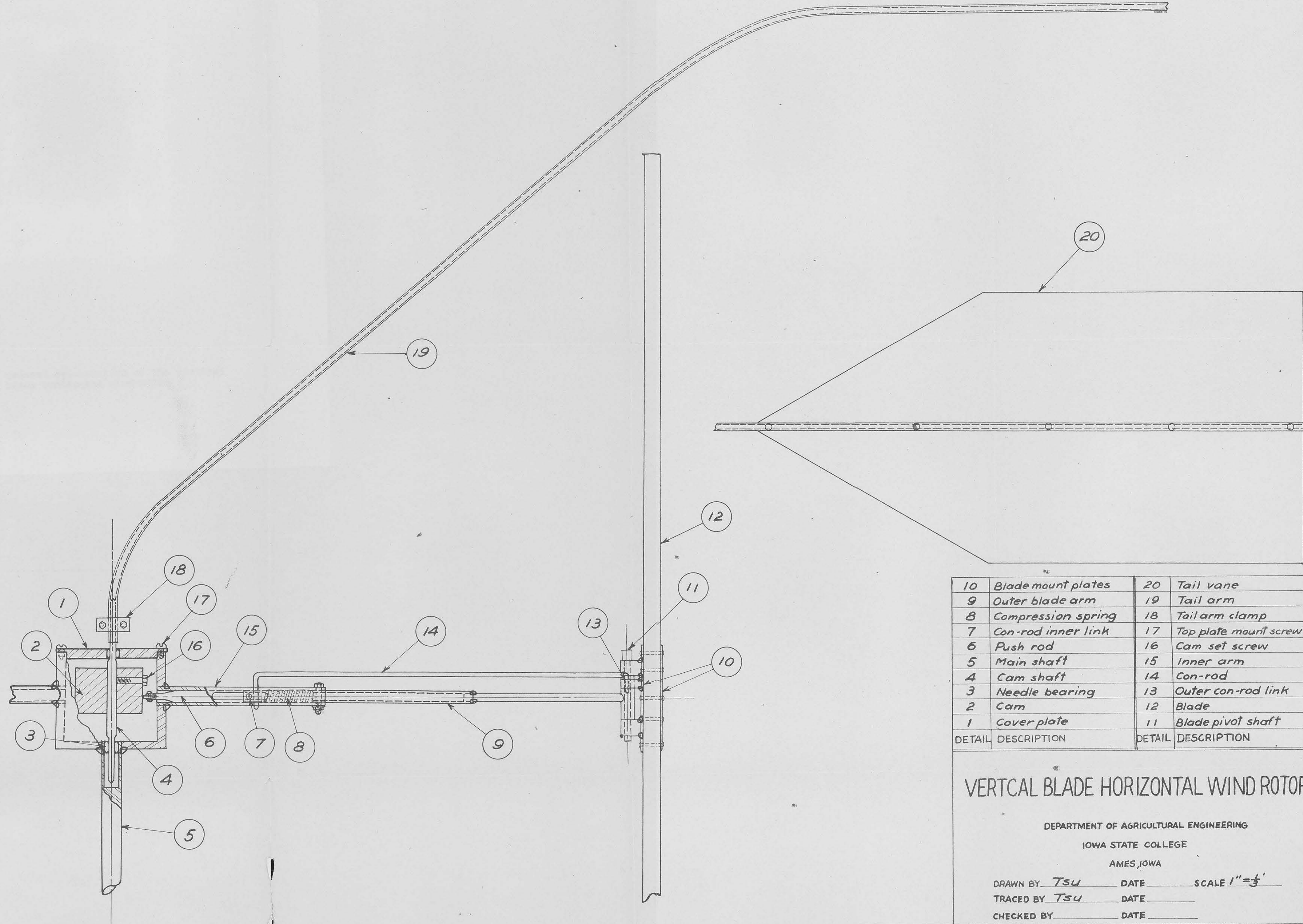
The Chinese windmill closely resembles Kirsten's cycloidal propulsion except that the position of the vanes is not accurately controlled so that the relative wind will not hit the vanes with the best angle of attack for power generation. The interesting point is that, by a principle similar to that by which a boat proceeds against the wind, the sail furnishes power through much more than one-half of a revolution.

One similar machine has been built by Donaldson (25). His machine has the same arrangement of wings and the main shaft except that the wings are true airfoils and the wing regulating mechanism consists of rods linked to a crank pin governed by a directional vane. As the wind blows on the windmill the directional vane pulls the crank pin to the leeward side and gives the wings on either side of the main shaft a symmetrical pivot motion. Donaldson claimed that his windmill had 46 percent efficiency as against 21 percent for a propeller type.

There are also other types of windmills such as Jumbo and Pantanemone types (16). They are, however, rather more curious than useful and will not be discussed here.

III. DESIGN AND CONSTRUCTION

The windmill designed by the author is a testing model which consists of airfoil sections like aeroplane wings which are mounted around a vertical shaft on pivots that permit movement on either side of their circular path. The movement of the wings is controlled in such a way that the relative wind will hit the wings in any location along the circular path with a definite angle of attack which will give maximum lift or turning force to the main shaft. The control mechanism consists of connecting rods linked with wings which are actuated by spring mounted push rods with roller followers on a specially designed cam. As can be seen from Figures 9 and 10, since the direction of the relative wind at different points of the travelling circle is not symmetrical with respect to the direction of the absolute wind, Donaldson's simple crank mechanism will never accurately keep the wings in the desired position. In order to achieve the perfect regulation of the wings it would seem that cam control is the only



DETAIL	DESCRIPTION	DETAIL	DESCRIPTION
10	Blade mount plates	20	Tail vane
9	Outer blade arm	19	Tail arm
8	Compression spring	18	Tail arm clamp
7	Con-rod inner link	17	Top plate mount screw
6	Push rod	16	Cam set screw
5	Main shaft	15	Inner arm
4	Cam shaft	14	Con-rod
3	Needle bearing	13	Outer con-rod link
2	Cam	12	Blade
1	Cover plate	11	Blade pivot shaft

VERTCAL BLADE HORIZONTAL WIND ROTOR

DEPARTMENT OF AGRICULTURAL ENGINEERING

IOWA STATE COLLEGE

AMES, IOWA

DRAWN BY Tsu DATE ____ SCALE 1" = 1/3'

TRACED BY Tsu DATE ____

CHECKED BY ____ DATE ____

Fig. 8. Vertical blade horizontal wind rotor

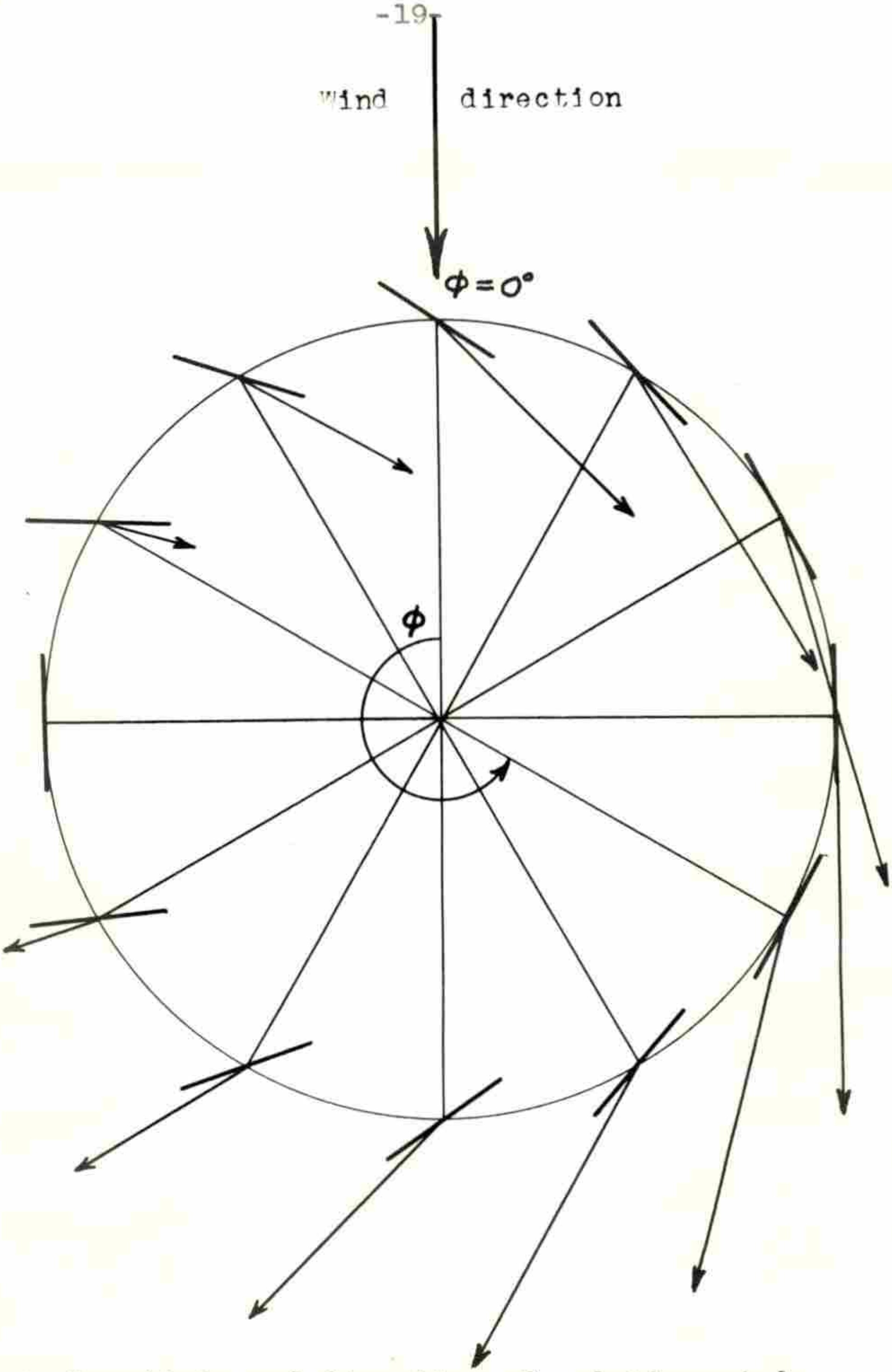


Fig. 9. Magnitude and direction of relative wind and position of blades. Assumed wind velocity, 22ft./sec., $V_B/V_A = 1$, scale 1 in. = 20 ft./sec.

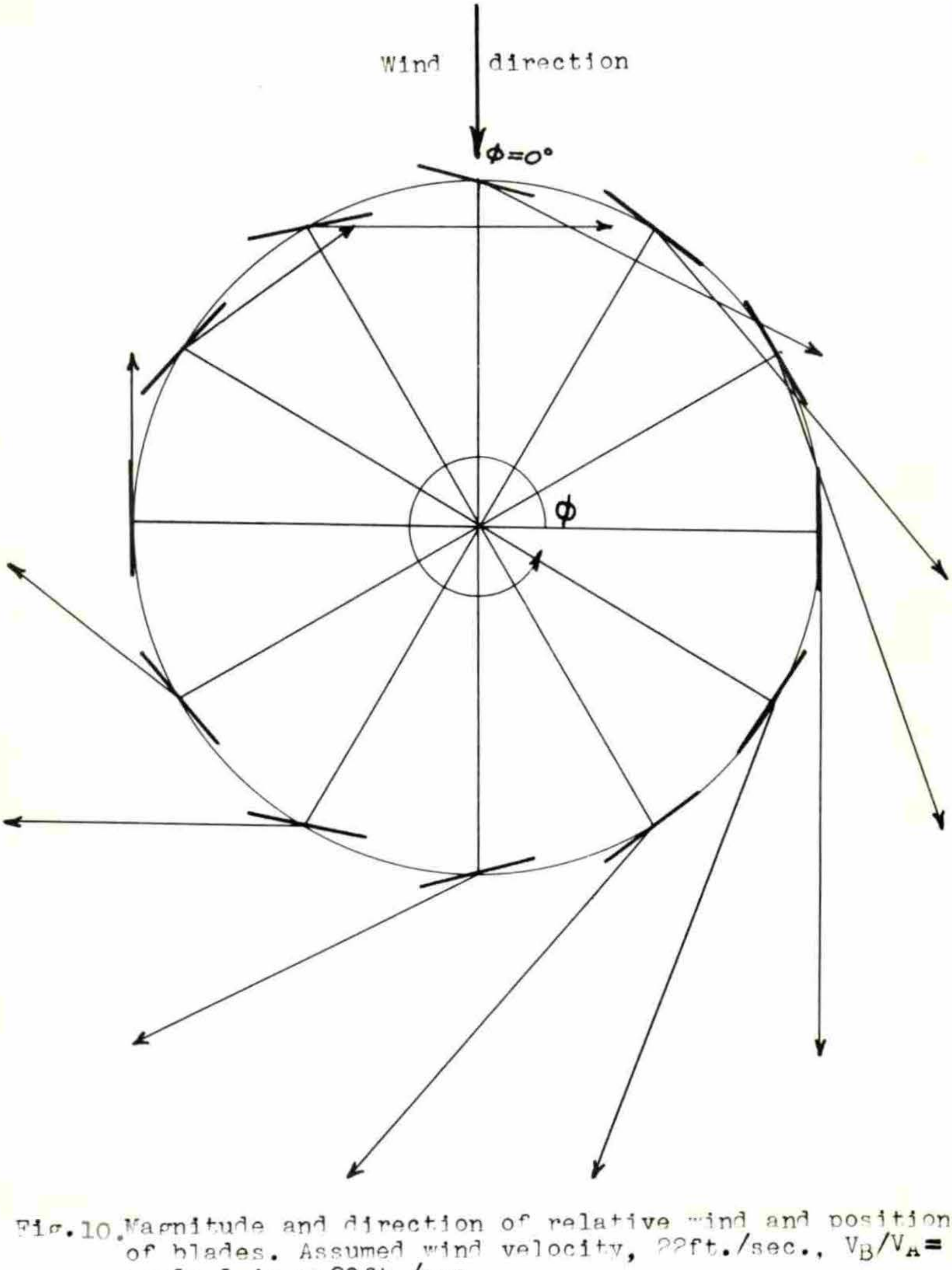


Fig.10. Magnitude and direction of relative wind and position of blades. Assumed wind velocity, 22ft./sec., $V_B/V_A = 2$, scale 1 in.= 20ft./sec.

alternative.

The general construction of the windmill is clearly shown in Figures 7 and 8. The only parts needing explanation are the wings and cam. They are fully discussed in the following paragraphs:

A. Wings

Because the wings are regulated so they will be hit by the relative wind on both sides (Figures 9 and 10), the wing section has to be symmetrical. The N. A. C. A. No. 0009 section is chosen for this purpose.

The maximum C_L and C_D of this particular airfoil both occur at 16 degrees of angle of attack. The curve of C_L against angle of attack rises slowly with increasing angle before 16 degrees according to the equation,

$$C_L = 0.075\alpha$$

where α is angle of attack in degrees. When the angle of attack becomes larger than 16 degrees, the curve drops suddenly. To avoid the sudden drop of C_L in case the angle of attack becomes greater than 16 degrees due to turbulent air stream and unavoidable inaccuracy in constructing the machine, the angle of attack is chosen as 14 degrees which will give a C_L of 1.05 and a C_D of 0.08.

There are altogether four wings on the windmill, each measuring 6 feet by 10 inches. Each of them is made of a piece of plywood sandwiched with casein glue between two pieces of red cedar sidings to give the desired strength and lightness. The wings are then hand-planed until true to shape.

The values of C_L and C_D are not corrected according to the aspect ratio and the effect of interference because:

1. The aspect ratio of the blade is 36/5 or 7.2 which is high enough to avoid any appreciable influence on the value of C_L and C_D.

2. The speed of the designed windmill will be slow as will be explained later, and the number of blades is low in comparison with other windmills of similar speed ratio between blade and wind such as the American 18-blade type. Not much interference is expected.

B. Cam

When the wing starts to revolve around the main shaft in a constant-speed wind, the speed of the wing is linearly constant but its direction of motion changes from 0 to 360 degrees in each revolution. As the relative wind velocity is the vector resultant of the absolute wind velocity (V_A) and the peripheral blade velocity (V_B), its magnitude and direction change all the way through each complete revolution. However, if the ratio of V_B/V_A is constant and V_B varies proportionally with V_A, the sense of the relative velocity at a certain point on the circular path of the blade will always be the same and will be repeated in each revolution. The most important thing in designing this type of windmill is to find the probable V_B/V_A so that a suitable cam can be made accordingly.

If the wind velocity is assumed to be 15 m.p.h. or 22 ft./sec., the turning force created on each blade at different locations along the circular path can be calculated in the following way:

1. Formula. From Figure 11 we learn that the turning force (F) is,

$$F = L\cos\epsilon - D\sin\epsilon$$

where ϵ is the acute angle formed by the relative wind and the arm,

L is the lift force,

D is the drag force.

Again $L = \frac{1}{2}C_L\rho AV_{\frac{A}{B}}^2$ and $D = \frac{1}{2}C_D\rho AV_{\frac{A}{B}}^2$

where ρ is the density of air which is 0.002378 slug/ft.[4]

A is the area of the wing in square feet,

and $V_{\frac{A}{B}}$ is the relative wind velocity ($V_{\frac{A}{B}} = V_A \rightarrow V_B$)

2. Calculation. Based upon the formula, the net turning force exerted by the wind on each blade at different locations is calculated according to $V_B/V_A = 1$ and $V_B/V_A = 2$ (Tables 4 and 5).

Table 4. Turning force exerted by 15 m.p.h. wind on one blade, velocity ratio = 1

Rotation angle	ϵ	$V_{A/B}$ ft/sec.	V^2	cos ϵ	sin ϵ	L Lb.	D Lb.	L cos ϵ	D sin ϵ	Net force
0	45	31	960	0.707	0.707	4.57	0.34	4.25	0.32	3.93
30	30	22	484	0.866	0.500	2.30	0.17	2.66	0.11	2.55
60	15	11	128	0.966	0.259	0.61	0.05	0.78	0.01	0.77
90*	0	0	0	0.000	0.000	0.00	0.00	0.00	0.00	0.00
120	15	11	128	0.966	0.259	0.61	0.05	0.78	0.01	0.77
150	30	22	484	0.866	0.500	2.30	0.17	2.66	0.11	2.55
180	45	31	960	0.707	0.707	4.57	0.34	4.25	0.32	3.93
210	60	38	1440	0.500	0.866	6.85	0.51	4.50	0.57	3.93
240	75	42	1760	0.259	0.966	8.37	0.62	2.84	0.80	2.04
270*	90	43	1874	0.000	1.000	0.00	0.00	0.00	0.00	0.00
300	75	42	1760	0.259	0.966	8.37	0.62	2.84	0.80	2.04
330	60	38	1440	0.500	0.866	6.85	0.51	4.50	0.57	3.93

* Angle of attack is made equal to zero to avoid negative force

Table 5. Turning force exerted by 15 m.p.h. wind on one blade, velocity ratio = 2

Rotation angle	ϵ	$V_{A/B}$ ft/sec.	V^2	cos ϵ	sin ϵ	L Lb.	D Lb.	L cos ϵ	D sin ϵ	Net force
0	63	49	2400	0.454	0.890	11.40	0.85	6.80	1.01	5.79
30	60	39	1520	0.500	0.866	7.23	0.54	4.75	0.63	4.12
60	66	27	730	0.484	0.874	3.47	0.26	2.21	0.29	1.92
90*	90	22	484	0.000	1.000	2.30	0.00	0.00	0.00	0.00
120	66	27	730	0.484	0.874	3.47	0.26	2.21	0.29	1.92
150	60	39	1520	0.500	0.866	7.23	0.54	4.75	0.63	4.12
180	63	49	2400	0.454	0.890	11.40	0.85	6.80	1.01	5.79
210	71	59	3480	0.325	0.945	16.55	1.24	7.05	1.55	5.50
240	81	65	4220	0.156	0.986	20.00	1.50	4.10	1.95	2.15
270*	90	67	4480	0.000	1.000	0.00	0.00	0.00	0.00	0.00
300	81	65	4220	0.156	0.986	20.00	1.50	4.10	1.95	2.15
330	71	59	3480	0.325	0.945	16.55	1.24	7.05	1.55	5.50

*Angle of attack is made equal to zero to avoid negative force

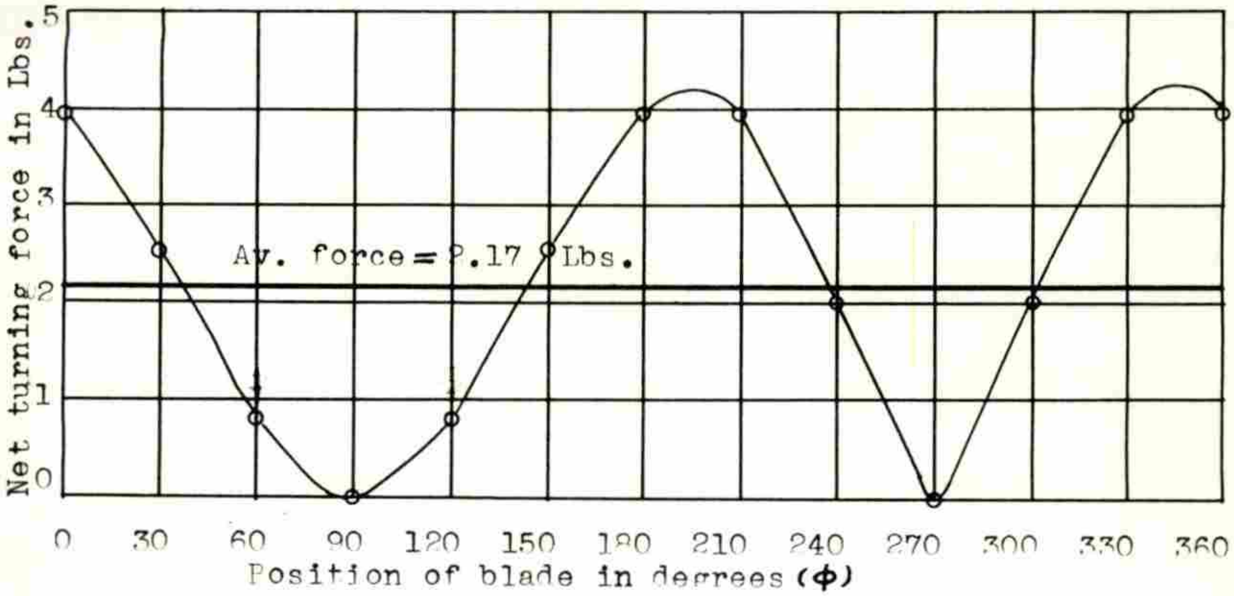


Fig.11. Net turning force exerted on each blade by 15mph wind. Angle of attack = 14°, $V_B/V_A = 1$

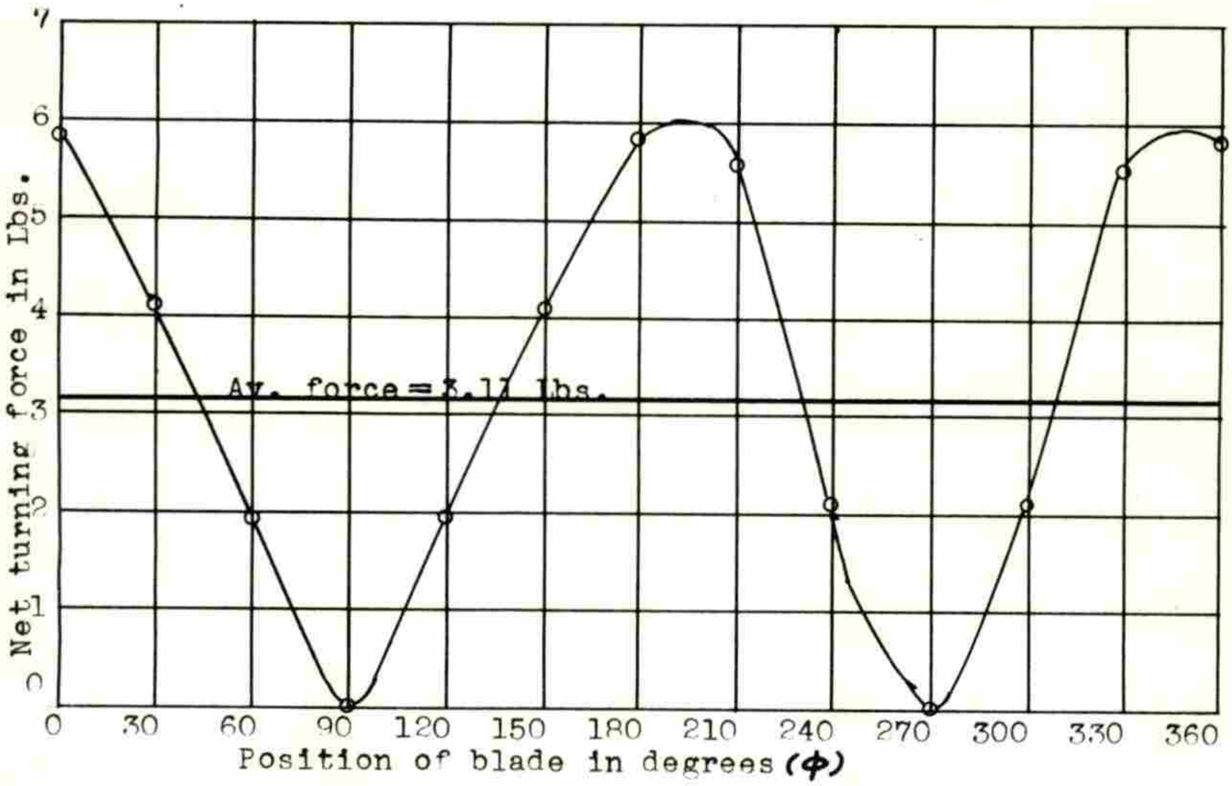


Fig. 12. Net turning force exerted on each blade by 15mph wind. Angle of attack=14°, $V_B/V_A = 2$

3. Plot force curves. The force curves are plotted in Figures 11 and 12. The areas under the curves are measured with planimeter and then divided by the abscissa to give the average force. With the angle of attack regulated at 14 degrees, the average net turning force exerted on each 6 ft. by 6 in. N. A. C. A. 0009 wing is found to be 2.17 lbs. for $V_B/V_A = 1$ and 3.11 lbs. for $V_B/V_A = 2$.

The horse power developed by one blade in a 15 m.p.h. wind is:

(A) $V_B/V_A = 1$,

$$Hp. = 22 \times 2.17/550 = 0.0868$$

(B) $V_B/V_A = 2$,

$$Hp. = 44 \times 3.11/550 = 0.249$$

From the above calculation we know that the horse power developed increases as V_B/V_A increases. This can be proved again by mathematical analysis.

Suppose the wind of velocity $A(V_A)$ is blowing in the direction shown in Figure 13, and the blade is in such position that the arm makes an angle of ϕ with the wind direction; by vector subtraction the relative velocity $(V_{\frac{A}{B}})$ is found as shown in Figure 14. Again from Figure 15 we learn that the net turning force is the resultant of the projections of lift(L) and drag(D) on the tangential line to the circular path at the blade pivot. In mathematical expression we have:

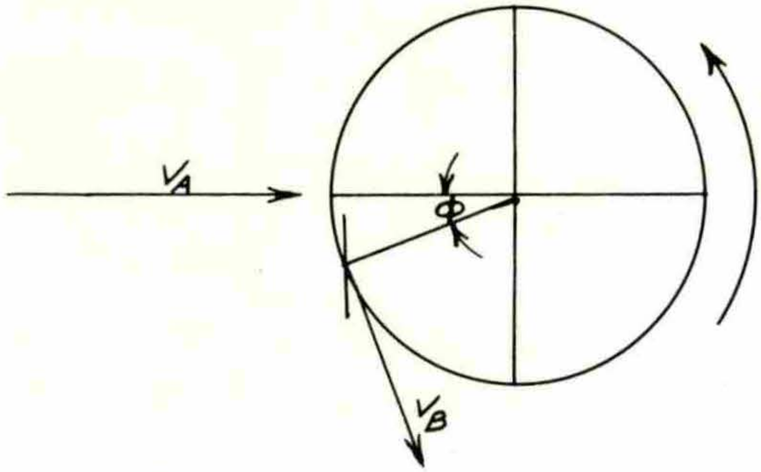


Fig.13.Diagram showing wind rotor arrangement and wind and blade velocities.

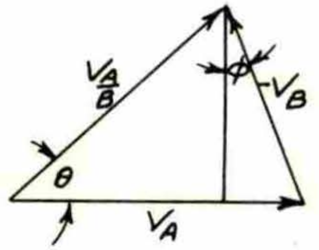


Fig. 14.Diagram showing vector substraction to get $V_{\frac{A}{B}}$

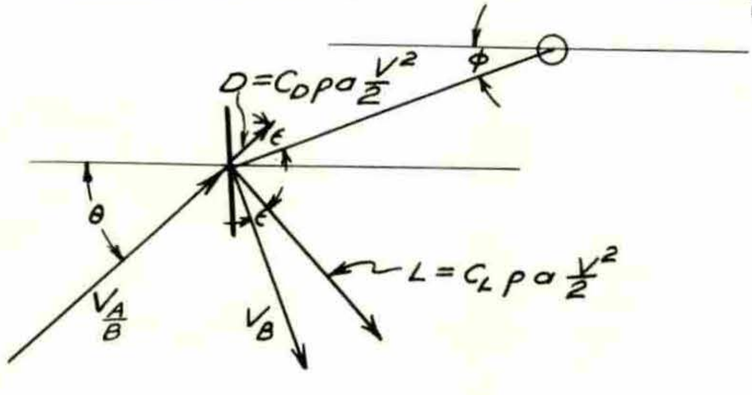


Fig.15. Lift and drag on one blade

$F = L\cos\epsilon - D\sin\epsilon$

$$= A\rho \frac{V_{A/B}^2}{2} (C_L\cos\epsilon - C_D\sin\epsilon)$$

where ϵ is the angle between relative velocity and the blade arm. Now if we call the angle between the relative wind velocity and the absolute wind velocity to be θ and the angle between absolute wind velocity and the blade arm to be ϕ, we learn from Figure 15 that:

$$\epsilon = 180^\circ - (180^\circ - \theta + \phi) = \theta - \phi$$

and from Figure 14,

$$\theta = \tan^{-1} \frac{V_B\cos\phi}{V_A - V_B\sin\phi}$$

Therefore,

$$F = A\rho \frac{V_{A/B}^2}{2} ((C_L\cos(\theta-\phi) - C_D\sin(\theta-\phi)))$$

$$= A\rho \frac{V_{A/B}^2}{2} ((C_L\cos(\tan^{-1} \frac{V_B\cos\phi}{V_A - V_B\sin\phi} - \phi) - C_D\sin(\tan^{-1} \frac{V_B\cos\phi}{V_A - V_B\sin\phi} - \phi))) \quad \ldots\ldots(A)$$

Again from Figure 14,

$$V_{A/B} = \sqrt{V_B^2 \cos^2\phi + (V_A - V_B\sin\phi)^2}$$

$$= \sqrt{V_B^2 \cos^2\phi + V_A^2 - 2V_AV_B\sin\phi + V_B^2 \sin^2\phi}$$

$$= \sqrt{V_B^2(\cos^2\phi + \sin^2\phi) + V_A^2 - 2V_AV_B\sin\phi}$$

$$= \sqrt{V_B^2 + V_A^2 - 2V_AV_B\sin\phi}$$

$$\sin\left(\tan^{-1}\frac{V_B\cos\phi}{V_A-V_B\sin\phi}-\phi\right)$$

$$=\sin\left(\tan^{-1}\frac{V_B\cos\phi}{V_A-V_B\sin\phi}\right)\cos\phi-\cos\left(\tan^{-1}\frac{V_B\cos\phi}{V_A-V_B\sin\phi}\right)\sin\phi$$

$$=\frac{V_B\cos\phi}{\sqrt{V_B^2+V_B^2-2V_AV_B\sin\phi}}\cos\phi-\frac{V_A-V_B\sin\phi}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}\sin\phi$$

$$=\frac{V_B-V_A\sin\phi}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}$$

$$\cos\left(\tan^{-1}\frac{V_B\cos\phi}{V_A-V_B\sin\phi}-\phi\right)$$

$$=\frac{V_A-V_B\sin\phi}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}\cos\phi+\frac{V_B\cos\phi}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}\sin\phi$$

$$=\frac{V_A\cos\phi}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}$$

Substituting those values in (A), we have:

$$F=1/2\,A\rho(V_B^2+V_A^2-2V_AV_B\sin\phi)\left(\left(\frac{C_LV_A\cos\phi}{\sqrt{V_B^2+V_A^2-2V_AV_A\sin\phi}}-\frac{C_D(V_B-V_A\sin\phi)}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}\right)\right)$$

$$= \frac{A\rho}{2}\left(\left(\frac{C_LV_AV_B^2\cos\phi - C_DV_B^3 + C_DV_AV_B^2\sin\phi + C_LV_A^3\cos\phi - C_DV_A^2V_B + C_DV_A^3\sin\phi - 2C_DV_A^2V_B\sin^2\phi - 2C_LV_A^2V_B\sin\phi\cos\phi + 2C_DV_AV_B^2\sin\phi}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}\right)\right)$$

$$= \frac{A\rho}{2}\left(\left(\frac{(C_LV_AV_B^2+C_LV_A^3)\cos\phi + (C_DV_AV_B^2+C_DV_A^3+2C_DV_AV_B^2)\sin\phi - C_DV_B^3 - C_DV_A^2V_B - 2C_LV_A^2V_B\sin\phi\cos\phi - 2C_DV_A^2V_B\sin^2\phi}{\sqrt{V_B^2+V_A^2-2V_AV_B\sin\phi}}\right)\right)$$

Express in terms of V_B/V_A or R,

$$F = A\rho VA^2\left(\left(\frac{(C_LR^2+C_L)\cos\phi + (C_DR^2+C_D+2C_DR^2)\sin\phi - C_DR^3 - C_DR - 2C_LR\sin\phi\cos\phi - 2C_DR\sin^2\phi}{\sqrt{R^2+1-2R\sin\phi}}\right)\right)$$

From the force curves (Figures 11 and 12) we know that the maximum turning force always happens when ϕ is equal to 0^o or 180^o no matter what is the value of V_B/V_A. Hence, the optimum value of V_B/V_A which gives maximum force at those locations will also give maximum developed power. ϕ is set = 0^o.

$$F = A\rho V_A^2\left(\left(\frac{C_LR^2+C_L-C_DR^3-C_DR}{\sqrt{1+R^2}}\right)\right)$$

$$\frac{\delta F}{\delta R} = \sqrt{1+R^2}(2C_L R - 3C_D R^2 - C_D) - (C_L R^2 + C_L - C_D R^3 - C_D R)(1+R^2)^{-\frac{1}{2}} R = 0$$

Solving the above equation, we get R = 6.4 for maximum force. However, the calculation of the efficiency indicates that the value of V_B/V_A will never reach the optimum value of 6.4. The total power contained in a stream of 15 m.p.h. moving air with a sectional area equal to that of the blade area (6 ft. by 10 in., or 5 square feet) is:

$$Hp = 1/2MV^2/550$$

$$= (1/2)(5)(22)(0.002378)(22)^2/550$$

$$= 0.115$$

The efficiency (E) of one blade with $V_B/V_A = 1$ is,

$$E = 0.0868 \times 100/0.115 = 75.5\%$$

and the efficiency of one blade with $V_B/V_A = 2$ is,

$$E = 0.249 \times 100/0.115 = 217\%$$

Since it is impossible to get an efficiency higher than 100%, we can conclude here that the value of V_B/V_A will not even reach 2, although the higher the value of V_B/V_A , the higher will be the efficiency; the highest speed ratio seems to lie in the neighborhood of 1.

Based upon the above findings, only cams designed according to speed ratios 1 and 2 are used for experimental purposes (16).

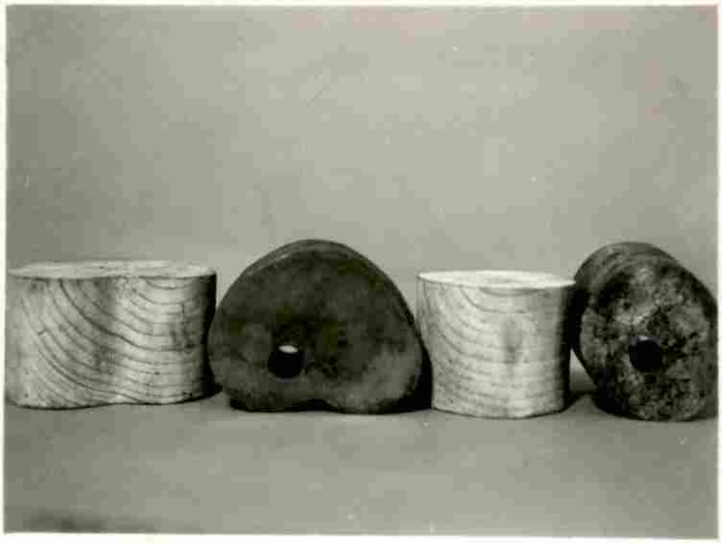


Fig. 16. Patterns and cams of velocity ratios 1 and 2

IV. TEST OF THE DESIGNED WINDMILL

The windmill with the vertical blades regulated by the spring-mounted followers on a regular solid cam was constructed according to the design drawing (Figure 7). Four weakest springs were used for the four followers, yet they were strong enough to give the blades a quick return motion. The spring modulus was found to be 6 lbs. per inch of compression. Two cast iron cams designed according to velocity ratios 1 and 2 were separately used in the tests. The dynamometer consists of a brake wheel with an 8-inch arm hitched to a 50-pound dairy spring scale. It was directly mounted on the main shaft of the windmill. Before testing the machine the required positions of the blades were checked position to position to make sure that the machine was built reasonably accurate. The whole outfit was then mounted on a trailer (Figure 8) and pulled by a car on the paved highway on windless days. It was assumed that in this case the speedometer readings of the car would represent the speed of the wind hitting the blades. The result of the tests and computations are tabulated in the following:

1. Windmill with 2:1 velocity ratio cam

The windmill of this arrangement started to run in a 12 m.p.h. wind. A test was made by having the car pulling steadily at 15 m.p.h. Testing data are recorded in Table 6.

Table 6. Testing data of the windmill with 2:1 velocity ratio cam

Load in lbs.	Rpm	Ft.-Lbs./Min.	Hp. developed	Efficiency percent
0	44	0	0	0.00
4	36	576	0.018	2.24
6	24	576	0.018	2.25
10	18	720	0.022	2.79
12	16	770	0.023	2.99
16	4	256	0.008	1.00

In Table 6 the efficiency was calculated by taking the power contained in the 15 m.p.h. air stream as the total input which is:

$$\text{Power input} = \frac{1}{2}MV^2$$

$$= \frac{5.67(6)(22)^3(0.002378)}{2(550)}$$

$$= 0.782 \text{ Hp.}$$

The extremely low efficiency is due to two reasons:

a. For a windmill of 5 ft. and 8 in. diameter revolving in a 15 m.p.h. (or 1,320 ft./min.) wind, the blades have to travel at a speed of 2,640 ft. per min. to reach the velocity ratio of 2:1. The revolutional speed, in this case, should be 148.3 rpm. As the revolutional speed in this test has never been over 50, the direction of the relative wind deviated so much from the desired direction that the machine would never operate efficiently.

b. This machine, due to the heavy pressure exerted by the spring-mounted followers on the cam, has comparatively high frictional loss of energy. It was found roughly that the torque needed to turn the windmill revolving slowly under no load condition was 10 Ft.-Lbs., or 15 lbs. force at a radius of 8 inches. The efficiency could be doubled if the friction could be reduced to approaching zero.

2. Windmill with 1:1 velocity ratio cam

This arrangement enables the windmill to start at a lower wind velocity. It started in 8 m.p.h. wind. A test was made with the trailer moving at 15 m.p.h. speed. The results are tabulated in Table 7.

Table 7. Testing data of the windmill with 1:1 velocity ratio cam

Load in lbs.	Rpm	Ft.-Lbs./Min.	Hp. developed	Efficiency percent
0	52	0	0	0.00
2	42	336	0.012	1.31
4	40	640	0.020	2.48
8	36	1150	0.035	4.46
10	33	1320	0.040	5.12
12	28	1345	0.041	5.21
14	10	555	0.017	2.17

The efficiency in Table 7 is again calculated by taking the power contained in the 15 m.p.h. air stream as the total input. As we can see from Table 7 the efficiency of this arrangement is still very low. The frictional loss in this kind of arrangement is even much more than in the first arrangement, because of the increased cam displacement which causes more compression of the springs. The measured torque necessary to keep the windmill turning slowly in still air is roughtly 19.34 ft.-lbs. or 29 lbs. at 8-inch radius. If the frictional loss could be reduced to near zero, and the friction in this machine is assumed to be quite constant in different rotational speeds, the efficiency can be calculated much higher as shown in Table 8.

Table 8. Calculated efficiency, assuming the frictional loss being convertible into useful work

Load in lbs.	Load plus friction lbs.	Developed and frictional Hp. together	Calculated efficiency %
0	29	0.183	23.4
2	31	0.158	21.0
4	33	0.160	20.4
6	35	0.161	20.6
8	37	0.161	20.6
10	39	0.156	19.9
12	41	0.139	17.8
14	43	0.052	6.7

The fastest rotational speed in this test with 1:1 cam is 52 rpm or 5.67(3.14)952) = 925 ft./min. in linear speed. The speed of the 15 m.p.h. wind is 1320 ft./min. The real velocity ratio in this case is 925/1,320 or 0.7 which is lower than the cam design ratio of 1. However, the rotational speed can be expected to increase with the load decreased. The efficiency will, consequently, increase too.

V. DISCUSSION

A. THEORETICAL EFFICIENCY

The theoretical efficiency of a typical windmill with its rotational plane normal to the wind direction has been calculated by many mathematicians using different methods of attack. The Glauert's theory (5), however, seems to be the most widely accepted. The calculated theoretical maximum efficiency is 59.3 percent of the energy contained in the wind.

It is evident from the Figures 11 and 12 that the maximum turning force created on a blade of the vertical blade type of windmill happens when the angle of rotation ϕ is near 0° or 180°. This same blade would produce the same turning effect if it were mounted on an American type of windmill with "similar velocity ratio"* and subjected to the same wind velocity. The turning force of one blade on the vertical blade type of windmill will, however, diminish to zero whenever the rotational angle ϕ approaches 90° or 270°, while that of one blade on the American type will stay constant throughout the complete revolution.

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*The velocity ratio of the blade element at the mean radius

Disregarding the wind interception area, the power generated by the two different types of windmills having similar blades will be in the ratio of the maximum ordinate of the force curve in Figure 11 to the mean ordinate which is 3.93: 2.17 or 1.81:1. In other words, the American type windmill will generate 1.81 times more power per blade than the vertical blade type. However, the American type having blades six feet long will intercept a larger area of wind than the vertical type having blades of the same length. The ratio is $\pi 6^2 : 34 = 113 : 34 = 3.32$. This means that if both windmills have only one blade and have the same velocity ratio, the vertical type will have an efficiency 3.31:1.81 or 1.83 times the efficiency of the American type. Moreover, due to the difficulty of shaping and spacing of the blades near to the center of rotation, the American type as a rule does not have the central part of the vanes functioning and thus sacrifices at least 30 percent of its efficiency. Now the efficiency ratio of the two different types of windmills becomes 1.83: 0.7 or 2.61:1 in favor of the vertical blad type.

When the number of blades increases on both windmills, the situation becomes quite different. In the vertical blade type the blades behind the front blades will never be hit by the wind effectively. For a windmill containing four vertical blades, the first blade at the position of $\phi = 0^o$ is directly in front of the third blade, and the second and the fourth blades will not produce any power due to zero angle of attack. The number of effective blades in this case will be only one. When the first blade comes to the position $\phi = 45^{\circ}$, the first and the fourth blades will be directly in front of the second and the third blades. The number of the effective blades in this case will be only two. The total power output will be very much handicapped by this kind of interference, but on an American type of windmill the number of blades can be increased up to 16 or more without much induced interference.

Assuming that the four blades on a vertical blade type could operate perfectly without any interference at all, the American type having four times the number of blades of the same shape and size will easily offset the efficiency ratio of 2.61:1 originally in favor of the vertical blade type.

As the propeller type of windmill is beyond doubt more efficient than the American type (14), it seems to be very doubtful that the theoretical efficiency of a vertical blade horizontal rotor type could be higher than the propeller type.

B. Starting Torque

The starting torque depends upon the pitch angle of the blades. The propeller type does not have a good starting torque because of its small pitch angle of about two to four degrees at the blade tips. When the wind starts to hit the blade at an angle of attack of around 87 degrees, which is at least 70 degrees off the optimum angle of attack, the lift will doubtlessly be small. For a windmill with velocity ratio in the neighborhood of 1, the relative wind will make an angle of about 45 degrees to the rotational plane of the blades. The pitch angle will be in this case about 33 degrees, if we set the angle of attack at 12 degrees. When the wind starts to hit the stationary blade, the initial angle of attack will be much smaller and nearer to the optimum angle of attack than the propeller type and thus will give a larger lift and consequently a stronger starting torque.

As most airfoil data are confined to the angles of attack up to 30 degrees, there is no means to calculate the starting torque of a windmill because of the large initial angle of attack. As a rough estimation we shall assume the airfoils to be flat plates and the force exerted by the wind to be normal to the blades. Based on Bollay's formula* and his experimental results (14), Table 9 is prepared to give the starting turning force of one plate on the vertical blade type windmill.

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*$F = C_N (1/2) V^2 S \rho$ where F is the normal force, C_N is the shape factor and S is the surface area. F multiplied by the cosine of the angle λ made by the directions of the blade motion and F will give the turning force.

Table 9. Starting turning force of one blade of the vertical blade type windmill

Blade position: ϕ degrees	Initial angle of attack: degrees	C_N	Normal force to blades lbs.	Angle λ between blade motion and F (degrees)	Starting turning force lbs.
300 or 240	29	0.98	2.82	89	0.05
330 or 210	44	1.00	2.88	74	0.79
0 or 180	59	1.12	3.22	59	1.63
30 or 150	74	1.18	3.40	44	2.44
60 or 120	80	1.18	3.40	29	2.98

It is evident from Figure 9 that the best starting position of the vertical blade type windmill is to have one of the four blades situated at $\phi = 60^o$ or 120^o. If one of the four blades is situated at $\phi = 60^o$, the rest of the blades will be at $\phi = 150^o$, 240^o, and 330^o. The total turning force will be, according to Table 9, 2.98+2.44+0.05+0.79 or a total of 6.62 lbs. in a 15 m.p.h. wind.

For an American type of windmill of the same wind interception area (34 sq. ft.), the conventional total blade area will be 7/8 of the total interception area or about 30 sq. ft. (7). The radius r will be:

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{34}{3.14}} = 3.28 \text{ ft.}$$

and its circumference c:

$$c = 2\pi r = 6.28(3.28) = 20.6 \text{ ft.}$$

The central part not covered by the blades will be 1/8 of

34 sq.ft. or 4 sq. ft. This left-over area will have a radius $r' = \sqrt{\frac{4}{\pi}} = 1.13$ ft. and a circumference $c' = 2\pi r$ $= 6.28(1.13) = 7.1$ ft. Hence the length of the blades will be $3.28-1.13 = 2.15$ ft. If this windmill has 18 vanes, the mean width will be $(7.1\ 20.6)/2(18) = 0.77$ ft. The mean aspect ratio will be $2.15/0.77 = 2.79$.

Actually, the blades on an American type windmill have a twist in their length. The central part of the blade has a larger pitch angle than the outer part and is, therefore, more favorable for starting. For comparison with the vertical blade type, we shall assume that the pitch angle of the blades on an American type windmill is uniform throughout their entire length. Then the initial angle of attack to this blade designed on velocity ratio of 1 will be like that of one blade on the vertical blade type at the position of $= 0^{\circ}$ or 180°. It will also be approximately 59 degrees. Calculated as flat plate again, the turning force T is:

$$T = C_N(1/2)V^2\rho S\cos\lambda$$

$$= 1.1(1/2)22^2(0.00238)34\cos 59^{\circ}$$

$$= 11.1 \text{ lbs.}$$

Therefore, the starting force of the vertical type windmill is much weaker than the American type.

C. Thrusting Force

The thrusting force T exerted by each blade on the bearings of the main shaft will be along the arm of the blade. Its magnitude is:

$$T = L\sin\epsilon + D\cos\epsilon.$$

Theoretically, the value of T will reach its maximum when ϕ reaches 270°. At this position the relative velocity of the wind hitting the blade is in its maximum magnitude and $\sin\epsilon$ is equal to 1 (Figure 9). However, this maximum value of T will never be reached, but on the contrary will be zero because of the feathering of the blades at this position. Similarly, the blade at the position of $\phi = 90°$ will not have any thrusting force on the main shaft bearings. If the windmill carries four vertical blades evenly spaced and has the first blade in the position of $\phi = 0°$, the only blades exerting thrusting force on the main shaft bearings will be the two at $\phi = 0°$ and 180°. Its magnitude is:

$$T = L_1\sin\epsilon_1 + D_1\cos\epsilon_1 + L_2\sin\epsilon_2 + D_2\cos\epsilon_2$$

From Figure 9 we know,

$$\epsilon_1 = \epsilon_2 = 45° \text{ and } V_{\frac{A}{B1}} = V_{\frac{A}{B2}}$$

Therefore,

$$\begin{aligned} T &= 2(L\sin\epsilon + D\cos\epsilon) \\ &= 2(0.707)(L+D) \\ &= 1.414(L+D) \end{aligned}$$

The sense of the force will be along the arms of the first and the third blades. By similar computation, Table 10 is prepared to show the combined thrusting force on the main shaft when the blades are in different positions.

Table 10. Variation of resultant thrusting force on the main shaft by blades at different positions. $V_B/V_A = 1$, wind velocity = 15 m.p.h.

Blade position of 1st blade in degrees	Magnitude lbs.	Sense, degrees from V_A in the third quadrant	Notes
0	7.23	0	Angle of attack of 4th blade=0^o
30	11.8	80	
60	11.5	18	
0	12.9	55	Angle of attack of 4th blade=0^o

From Table 10 we learn the following facts:

1. The maximum thrusting force of the designed windmill in a 15 m.p.h. wind is 12.9 lbs.

2. Unlike the ordinary windmills, the direction of the thrusting force changes all the time even when the windmill is subjected to a steady uniform wind.

The formula for calculating the thrusting force on an American or propeller type of windmill is given by Fales (14) as $T = 2\pi R^2 \rho V^2 a(1-a)$

where a is the wind velocity retarding factor which should be 1/3 for best efficiency. For an ordinary windmill having a wind interception area of 34 sq. ft. as our vertical blade type, the radius R, as we have calculated before, should be 3.28 ft. The velocity V is still assumed to be 15 m.p.h. or 22 ft. per second. Substituting all the known values into the equation, we get the value of T as 17.28 lbs. This value is comparatively higher than what we have calculated for the vertical blade type. However, this thrusting force will not change direction and magnitude in a steady uniform wind as the vertical blade type does. Consequently, we should use a much higher safety factor in designing a tower for the vertical blade type windmill.

VI. CONCLUSIONS

Since the maximum theoretical power obtainable from a given wind is about 6/10 of the total kinetic energy of the wind, it appears that with a windmill of 70 percent aerodynamic efficiency (14) and 90 percent gearing, 37 percent of the wind kinetic energy is the top limit to be expected for any practical application. It has already been shown in the previous discussion that the efficiency of the vertical blade type of windmill will not surpass this limit. The best efficiency tested on the model built to run at the velocity ratio of 1 is 23 percent on the assumption that the frictional loss could be cut down to near zero. This efficiency could be expected to increase if we can make the blades revolve faster by minimizing the friction.

However, the principle of designing a windmill is different from designing a water wheel. In water power plants, there is generally a very limited amount of energy available, which is extracted by relatively expensive power plants when the energy in the turbine is converted into useful form. It is very important for it to be done with the maximum efficiency, since the conversion process means a corresponding waste of the costly energy. The large capital investment and the interest on it also plays an important role. With windmills we have at hand such immense quantities of energy in the great ocean of air that we cannot possibly use it all. It is relatively unimportant as to how much energy is lost in the process of transformation, the cost being the only important thing. The best economical windmill is the one which furnishes the horse-power at the lowest figure. Therefore, in designing a windmill with limited construction materials, we should make the wind interception area as large as possible so that more power can be extracted from the air. From economical and practical points of view, the idea of arranging the blades in the form of a vertical blade horizontal rotor instead of the propeller form is entirely erroneous. Assuming the vertical blade type of windmill has the same overall efficiency as the propeller type, the latter, although having the same shape and form of blades as the former, will have a wind interception area three times that of the former and, thus, will extract three times more power than the former. Besides, the complication of the vertical blade type of windmill will increase the building and the maintenance costs. In conclusion, we shall say that the vertical blade horizontal wind rotor is hardly practical.

VII. SUMMARY

The vertical blade horizontal wind rotor is a reversed application of the Kirsten's cycloidal propulsion mechanism. The general construction is clearly shown in Figure 7, page 17. By cam control in the central hub, the vertical blades are regulated to always be hit by the relative wind with a best angle of attack of 14 degrees. As far as theoretical efficiency, starting torque, and thrusting force on the windmill tower are concerned, this type of windmill is hardly superior to the American type. On the other hand, the complication of its structure greatly handicaps its applicability. One model with four 6-foot blades mounted on four 2 ft.-10 in. arms built by the author was tested to have an efficiency of only 23 percent with the blades travelling at 7 tenths of the wind velocity. The optimum blade-wind velocity ratio for maximum efficiency was found to be in the neighborhood of 1.

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